Height of a Random Binary Tree

Masaaki Sibuya*, *Keio University, Yokohama, Japan*
Yoshiaki Itoh, *Institute of Statistical Mathematics, Tokyo, Japan*

A binary tree is a finite set of nodes, which is partitioned, if not empty, into a triple, root, left and right subtrees, where the root is a subset of single node and the subtrees are binary trees. In a random binary tree with n nodes, the cardinal number k of the left subtree is uniformly distributed on the integer interval $[0, n-1]$, and the both subtrees are random binary trees, where the partitioning random variables of the subtrees are conditionally independent. Random binary trees appears in quicksort and binary search trees (D.E. Knuth, 1973, The Art of Computer Programming, Vol. 3).

Let $F(h; n)$ denote the probability that the height of a random binary tree with n nodes is less than or equal to h. It satisfies the recurrence formula

$$F(h; n) = \frac{1}{n} \sum_{k=0}^{n-1} F(h-1; k) F(h-1; n-k-1),$$

$F(h; 0) = F(h; 1) = 1$ for $h \geqslant 0$, and $F(0; n) = 0$ for $n \geqslant 2$.

It is shown that $F(4 \log n; n) \to 0$ as $n \to \infty$. Some numerical values suggesting further asymptotic behavior of F are reported.

Nuclear Space Valued SDE's and Neuronal Applications

R.L. Wolpert, *Duke University, Durham, NC, USA*

Motivated by certain problems in neuroelectrophysiology, several infinite-dimensional stochastic differential equations and related martingale problems are introduced. Existence and uniqueness of weak solutions (and, under more stringent conditions, strong solutions) to these equations are probed using the Galerkin method of finite-dimensional approximation, introduced by Krylov and Rozovskii. The solutions are shown to lie in fixed Hilbert subspaces of Φ' for finite time intervals, and to obey maximal L^p bounds in the norms of those spaces.

2.3. Stochastic methods in physics

On Stochastic Quantization of Euclidean Quantum Field Theories

Ph. Blanchard, *University of Bielefeld, FR Germany*
J. Potthoff,* *Nagoya University, Japan, on leave of absence from Technical University of Berlin*
R. Sénéor, *Ecole Polytechnique, France*